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Instanton induced chiral symmetry breaking in extended QCD model

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Abstract. A mechanism for instanton induced chiral symmetry breaking in an extended QCD model (QCD with fundamental scalars) is proposed to describe quarks and gluons inside a baryon. The model Lagrangian that we use has the same symmetry properties as QCD. The scalar fields are shown to develop vacuum expectation values in the instanton background and generate masses for the three generation of quarks. The minimization condition is also used to break the flavour symmetry to make the s-quark heavier that the u and d quarks.

I Introduction

The mechanism of chiral symmetry breaking in QCD and consequent mass generation for quarks is an important issue. In the absence of explicit mass terms for the quarks, the Lagrangian for QCD has exact chiral symmetry $SU(N_f)_L \times SU(N_f)_R$, in addition to $U(1)_V$ (baryon number conservation) and $U(1)_A$ which, as is well known, is violated by quantum effects. Apart from the issue of the non-trivial nature of the QCD vacuum with the quark condensate $< 0|\bar{q}q|0 > \neq 0$, the nature of the chiral phase transition in QCD at finite temperature has been studied in [1] as well as in [2] and [3] making use of instanton type non-perturbative fluctuations of the gluon field. These studies revealed the fact that instanton type fluctuations of the gluon field are dominant in the QCD vacuum. At zero temperature, the effect of instantons in the QCD vacuum and in particular, understanding the nature of chiral symmetry breakdown has been initiated by 't Hooft [6] as also in [5], and [7] and the related earlier work of Raby [4]. The work of these authors was extended by Caldi [8] who showed that the instanton induced multiquark interaction i.e. the "effective interaction" found by 't Hooft for a colour $SU(2)$ gauge theory with N flavours of massless quarks, produces a spontaneously generated quark mass in a Hartree-Fock treatment. Subsequently, Dyakanov and Petrov [9] proposed a mechanism of spontaneous chiral symmetry breakdown based on the delocalization of zero fermionic modes of the Dirac operator in the presence of instantons. Here the Pauli-Villars regularization introduces an effective momentum-dependent mass $M(p)$ which is identified with the quark effective mass. The bare mass m for the quark, although introduced in the beginning, is taken to be zero eventually. Subsequently, in [10] an effective Lagrangian was used at low momenta of the σ model type involving quarks with a momentum dependent mass interacting with the chiral field to calculate the nucleon mass and nucleon σ term. In the studies of references [1] (on the nature of the chiral phase transition) and [4] (on the phenomenogical aspects of the η-decay width) an "effective theory" with the desired properties *has been assumed*. The usual practise followed by these authors to break the symmetry is to *add* a term $c(\det M + \det M^\dagger)$, where M is a colour singlet complex $N_f \times N_f$ matrix.

In this paper, we study the issue of instanton induced chiral symmetry breaking in an interacting model of quarks and gluons along with fundamental scalars described by a colour-singlet $N_f \times N_f$ complex matrix, allowing the scalars to have only a kinetic energy term and Yukawa coupling with the quark fields.

We briefly motivate the model and outline how it differs from other mechanisms. The low energy behaviour of QCD still eludes explicit calculations. Effective theories like the Gell-Mann Levy linear σ-model are used to give a low energy field theoretic description of strong interaction. The low energy phenomenology involving the spectroscopy and static properties of the baryons prefer the presence of pions along with quarks and gluons as illustrated by the success of chiral bag models and non-relativistic quark models. At present there is no reasonable explanation from QCD as to how these pionic degrees of freedom might arise and how they might be incorporated in QCD. Recently however, the chiral sigma model with quarks substituted for nucleons [14] has found support as a reasonable description of the nucleon [15], strong interaction properties at finite temperature and baryon density [17,16,18] even at scales well above chiral restoration, as well as weak interaction properties [19]. It has also been shown that the chiral linear sigma model with quarks, when coupled to gluons can be asymptotically free [20]. Furthermore, lattice studies [17,16] indicate that the chiral sigma model with quarks reproduce QCD lattice results rather well at finite temperature with the pions and sigma being elementary for all $T > T_\chi$ except not as Goldstone bosons. However gluons were neglected in these references.

In this paper, the model Lagrangian consists of the usual QCD Lagrangian and colour singlet scalar multiplet, has $SU(N_f) \times SU(N_f)$ flavour symmetry which entails taking $\sigma = \sigma^a \lambda^a, \pi = \pi^a \lambda^a$ *collectively* denoted by a complex $N_f \times N_f$ matrix M (where λ^a's are the usual Gell-Mann matrices) and is a possible candidate theory for strong interactions, in a phase of the theory (dictated by initial conditions on the QCD and Yukawa coupling and the renormalization scale μ) where not only the quarks but also the scalar and pseudoscalar mesons contained in M are elementary.

At this juncture we wish to also distinguish our approach from the many papers that have been written on this subject. The main differences with the references [1–3] is that we look at the zero temperature case and *do not* use the low energy effective Lagrangian of 't Hooft [6]. Instead, as we have explained earlier, our starting point is QCD with fundamental scalars in which it differs also from [9] in that we do not have a mass parameter to start with but is generated due to the Yukawa coupling. It is also different from [1] and [4] in that we do not *add* at the beginning $c(\det M + \det M^\dagger)$, but instead *generate* such a term explicitly from the zero modes of the Dirac operator by integrating out the quark fields in the instanton background. A similar approach to explain small fermion masses in QCD has been considered in [13] for a single component scalar field. The approach followed here is similar in spirit to [22] and the Nambu-Jona-Lasinio model motivated effective QCD of [11] and [12].

We describe the model in Sect. II and evaluate the quantum one-loop effective action with instantons as the classical background. In Sect. III the effective potential is obtained in the dilute gas approximation for the instantons. The one-loop effective potential is examined for a non-trivial minimum in Sect. IV and quark mass generation is studied. The results are summarized in Sect. V.

II The model

The partition function for the model explained in the introduction is

$$Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu\mathcal{D}M\mathcal{D}M^\dagger e^{i\int d^4x\mathcal{L}[\psi,\bar{\psi},A_\mu,M,M^\dagger]} \quad (1)$$

where ψ is the quark field, A^a_μ is the gluon field and M is a complex $N_f \times N_f$ matrix representing elementary scalars and pseudoscalars in the adjoint representation. The Lagrangian density in (1) is given by

$$\mathcal{L}(\psi,\bar{\psi},A_\mu,M,M^\dagger) = \mathcal{L}_{QCD} + +g_y(\bar{\psi}_L M\psi_R + \bar{\psi}_R M^\dagger\psi_L) + \frac{1}{2}\partial_\mu M\partial^\mu M^\dagger, \quad (2)$$

where $\mathcal{L}_{QCD}$ is the standard QCD Lagrangian with $\not{D} = \not{\partial} - ig\not{A}$, g_y is the Yukawa coupling strength for the interaction between M and ψ. $\mathcal{L}_{QCD}$ might contain the topological term $i\bar{\theta}F\bar{F}$ although we know that the experimental results on the neutron dipole moment set $\theta < 10^{-9}$. This Lagrangian density (2) possesses global $SU(N_f)_L \times SU(N_f)_R$ chiral invariance and of course, $SU(3)_c$ local color invariance. It has $U(1)_V$ (baryon number) and $U(1)_A$ invariances, the latter being broken by quantum corrections, leaving only a discrete $Z(N_f)_A$ symmetry. We evaluate the partition function, first by going over to Euclidean space and using the result that there exist, in QCD, non-perturbative gluon fluctuations *viz.* the instantons. We choose the instanton as the background field for gluons. The gauge fixing and ghost terms are to be introduced [6] and we suppress them here. They will be incorporated later. We first integrate over ψ and $\bar{\psi}$ fields to obtain

$$Z = \int \mathcal{D}A_\mu\mathcal{D}M\mathcal{D}M^\dagger e^{-\int d^4x[-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\theta\bar{F}F]} \det(i\bar{\not{D}} + ReM + i\gamma_5 ImM), \quad (3)$$

where we have expanded $A^a_\mu = \bar{A}^a_\mu(instanton) + a^a_\mu$ and retained $\bar{A}^a_\mu(instanton)$ terms only in the fermion determinant. Accordingly, $\bar{\not{D}} = \not{\partial} - ig\bar{\not{A}}$ (instanton) and the determinant above is over Lorentz and flavour indices. The functional integration over A^a_μ can be split over instanton locations $\bar{A}^a_\mu$(instanton)and and an integral over the fluctuations a^a_μ around the instantons. In the dilute gas approximation the Gaussian integral over a^a_μ gives a contribution [6] which we denote by K and the integration over $\bar{A}^a_\mu$ (instanton) is replaced by a summation over instanton winding numbers, apart from an overall Jacobian for the change of measure. In the evaluation of the fermion determinant, we first concentrate on the zero modes of the $\bar{\not{D}}$ operator, $(\gamma_5\psi_0 = \pm\psi_0)$. For each zero mode, we obtain $detM$ and $detM^\dagger$. The non-zero mode contribution gives $\det{}'(-\bar{\not{D}}^2 + MM^\dagger)$. The result of incorporating all of this is the partition function

$$Z = \int \mathcal{D}M\mathcal{D}M^\dagger \sum_{n_+}\frac{1}{n_+!}e^{in_+\theta}K^{n_+}(\det M)^{n_+} \sum_{n_-}\frac{1}{n_-!}e^{in_-\theta}K^{n_-}(\det M)^{n_-} \det{}'(-\bar{\not{D}}^2 + MM^\dagger)e^{-\int d^4x\partial_\mu M\partial^\mu M^\dagger} \quad (4)$$

where we have taken into account the exchange symmetry of the instantons. The summations over n_+ and n_- can be carried out and the effective action Γ defined through $Z = \int \mathcal{D}M\mathcal{D}M^\dagger e^{-\int\Gamma}$, can be written down to be

$$\Gamma = \frac{1}{2}\partial_\mu M\partial^\mu M^\dagger + K\det M + K\det M^\dagger + \ln\det{}'(-\bar{\not{D}}^2 + MM^\dagger) \quad (5)$$

where we have set $\theta = 0$.

To recapitulate, the above result shows that with the model proposed, the effect of the quantum one-loop calculations around an instanton background is to produce an effective action (5). This will be the starting point for the chiral symmetry breaking given in the following section. It is necessary to point out that this effective action is clearly different from the low energy effective actions employed in the studies of [8], [9] and [10]. To be explicit, in [8] the effective Lagrangian of 't Hooft [6] has been used which is different from what we have obtained above. In [9] and [10] although they do have the fermion determinant, the mass term is generated from the Pauli-Villars mass, a parameter in their approach. In our case, the Yukawa coupling of M with quark fields is responsible for the effective action (5). In fact, it is instructive to compare our effective action with that of [1] and [4]. The point we wish to emphasize is that the term $(detM + detM^\dagger)$ that these authors add is *derived* here as a contribution arising out of the zero modes of the Dirac operator in the instnton background. The importance of this term (see also [1] and [4]) is to break $U(N_f)\times U(N_f) \to SU(N_f)\times SU(N_f)\times U(1)_V$ which is the quantum symmetry of QCD. To be precise, the Lagrangian (2) has $G = SU(3)_c\times SU(N_f)\times SU(N_f)\times U(1)_V \times U(1)_A$ symmetries with the generators of G acting on ψ. By integrating out the quark fields, we obtain our effective action for M which has the same symmetry G with the generators now acting on M. In other words, the symmetry of QCD is preserved in our effective action. The role of the term $(detM + detM^\dagger)$ is obtained by considering $\theta \neq 0$, Then we would have got $Kcos\theta(detM+detM^\dagger)+iKsin\theta(detM-detM^\dagger)$. The second term violates CP and hence we have to put $\theta = 0$ consistent with the data on the electric dipole moment of the neutron.

The non-zero mode contribution in (5) is contained in $\det{}'(-\not{D}^2 + MM^\dagger)$. This term does not break $U(1)_A$ and will be a function of $MM^\dagger$. Following 't Hooft [6], we see that the regularised product of the non-vanishing eigen values of $(-\bar{\not{D}}^2 + MM^\dagger)$ is proportional to the result with $\bar{A}^a_\mu = 0$, the proportionality constant being dependent on the instanton quantum numbers (see (6.15) of [6]). Therefore we write this term as

$$A \ln \det{}'(-k^2 + MM^\dagger) \qquad (6)$$

where the constant A contains the effect of instantons taken to be compact in each small volume ΔV of space-time and also the ghost determinant arising from gauge fixing.

III One loop effective potential

We consider a basis in which the complex matrix M is diagonal with elements $\lambda_i, (i = 1, \ldots, N_f)$. This is done by considering the term $(\bar{\psi}_L M\psi_R + \bar{\psi}_R M^\dagger\psi_L)$ in (2). Diagonalization of the complex $N_f \times N_f$ matrix M, is done by independent left and right unitary transformations viz. $U_L^\dagger MU_R = \Lambda$ where $\Lambda_{ij} = \lambda_i\delta_{ij}$. This transformation, of course, introduces a mixing of flavours in ψ; nevertheless, the strong interaction part $\bar{\psi}\not{A}\psi (= \bar{\psi}_L\not{A}\psi_L + \bar{\psi}_R\not{A}\psi_R)$ is unaffected by this mixing. In general the diagonal elements λ_i's are complex. Using ϵ-regularizing scheme (see, for example, [21]) we get

$$\begin{aligned} A\ \ln\det{}'(-k^2 + MM^\dagger) &= A\ln\det{}'(-k^2 + \sum_i |\lambda_i|^2) \\ &\simeq A\sum_{i=1}^{N_f} |\lambda_i|^4 \ln(\frac{|\lambda_i|^2}{\mu^2}) \end{aligned} \qquad (7)$$

where μ is the regularization scale.

Combining all the results, the effective Lagrangian for M after diagonalisation becomes

$$\mathcal{L}_{eff} = \frac{1}{2}\sum_i \partial_\mu\lambda_i\partial^\mu\lambda_i^* + V_{eff}, \qquad (8)$$

where

$$V_{eff} = K\prod_{i=1}^{N_f}\lambda_i + K\prod_{i=1}^{N_f}\lambda_i^* + A\sum_{i=1}^{N_f}|\lambda_i|^4\ln(\frac{|\lambda_i|^2}{\mu^2}) \qquad (9)$$

Note that the first two terms above are a consequence of the topologically non-trivial instanton background for gluons that we have chosen. Such terms will not be present in a usual topologically trivial configuration like, for instance, the Saviddy background. This will prove to be crucial for our purposes.

IV Mechanism of chiral symmetry breaking

The minimum of the effective potential (9) for M will provide us with the vacuum expectation values for the diagonal elements of M. We restrict ourselves to $N_f = 3$. The minimisation with respect to $\lambda_1, \lambda_2, \lambda_3$ yields

$$\begin{aligned} K\lambda_2\lambda_3 &= -A\lambda_1^*|\lambda_1|^2(1 + 2\ln(\frac{|\lambda_1|^2}{\mu^2})) \\ K\lambda_3\lambda_1 &= -A\lambda_2^*|\lambda_2|^2(1 + 2\ln(\frac{|\lambda_2|^2}{\mu^2})) \\ K\lambda_1\lambda_2 &= -A\lambda_3^*|\lambda_3|^2(1 + 2\ln(\frac{|\lambda_3|^2}{\mu^2})) \end{aligned} \qquad (10)$$

from which, it immediately follows

$$\begin{aligned} |\lambda_1|^4(1 + 2\ln(\frac{|\lambda_1|^2}{\mu^2})) &= |\lambda_2|^4(1 + 2\ln(\frac{|\lambda_2|^2}{\mu^2})) \\ &= |\lambda_3|^4(1 + 2\ln(\frac{|\lambda_3|^2}{\mu^2})) \end{aligned} \qquad (11)$$

At this stage, the λ_i's are constants representing vacuum expectation values $< M >$. In general, the λ_i's are complex. Since however these are now space-time independent constants, their phases can be absorbed in U_L or U_R. With this provision, the λ_i's can be treated as real.

The trivial solution to (11) is of course $\lambda_1 = \lambda_2 = \lambda_3 = 0$ which corresponds to $V_{eff} = 0$. However, this solution set is not admissable since $\det M$ is not zero.

A non-trivial solution to (11) (corresponding to $\det M \neq 0$) is given by

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda \neq 0 \tag{12}$$

where

$$\lambda = \mu e^{-(1+\frac{K}{A\lambda})/4} \tag{13}$$

The value of the effective potential is then

$$V_{eff}(\lambda_1 = \lambda_2 = \lambda_3 = \lambda) = -2A\lambda^4(1 + \ln(\frac{\lambda}{\mu})) \tag{14}$$

We require this to be lower than $V_{eff} = 0$ (no scalar included) which is possible if

$$\mu < e\lambda \tag{15}$$

This sets the energy scale μ for which chiral symmetry breaking is possible. The symmetric solution (12) spontaneously breaks $SU(3)_L \times SU(3)_R$ to $SU(3)$ symmetry. Fluctuating M against the VEV in (12) and using (2), the symmetry breaking gives rise to masses for the quarks

$$m_u = m_d = m_s = g_y\lambda. \tag{16}$$

Thus, this model provides us with an explicit realization of chiral symmetry breaking through instantons, due to the presence of fundamental scalars. With $m_s \simeq 300$ MeV (effective strange quark mass, see, for example, the Particle Data Booklet), $g_y\mu < 600$ MeV which sets the scale in this model for chiral symmetry breaking.

The minimization condition also provides us with a more phenomenologically realistic scenario where all the quark masses are not equal. For this, we will assume that $\lambda_1 = \lambda_2 = \lambda \neq \lambda_3$. In particular we look for a solution for which $\lambda_3 = a\lambda$, such that

$$m_u = m_d \neq m_s \tag{17}$$

Substituting in the minimization condition (11), the result $\lambda_3 = a\lambda$, we get the following equation

$$\frac{a^4}{1-a^4}\ln a^2 = \frac{1}{2} + \ln(\frac{\lambda^2}{\mu^2}). \tag{18}$$

This equation is numerically solved for many representative values of a. In particular we choose values of $a = 20 - 40$ as the ratio of the s current quark mass to the u (or d) current quark mass. This immediately gives us a value of $\mu \simeq 26 - 27\lambda$ which for $g_y \sim 1$ gives $\mu \simeq 230$MeV, consistent with our previous estimate $\mu < 600$ MeV. We have broken the degeneracy between the quark masses between the s quark and the two lighter quarks, thereby breaking the $SU(3)$ flavour symmetry to isospin ($SU(2)$) and strangeness ($U(1)$). This kind of sequential symmetry breaking was envisaged long ago by Gell-Mann [22] by introducing $(\bar{\psi}_L M\psi_R + \bar{\psi}_R M^\dagger\psi_L)$ as an ideal pattern of couplings to "mesons". He added two types of terms; one is $u_0 \sim -m\bar{\psi}\psi$ which corresponds to, in our case $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ (16) and a term $u_8 \sim m_{02}(\bar{N}N - 2\bar{\lambda}\lambda)$ (in the notation of [22]) which corresponds to $\lambda_1 = \lambda_2 = \lambda; \lambda_3 \neq \lambda$, corresponding, in our case to (17). In this way, the effect of the u_0 and u_8 terms of Gell-Mann, which have been introduced later by many other authors (see, for example, [4]), can be reproduced from the solution (11) of the minimum of the effective potential in our model. With the addition of these terms, the study of the effective action (5) proceeds along the lines elucidated by Raby, by expanding $< M >$ around $< M >_0$.

V Conclusion

A mechanism for producing chiral symmetry breaking in QCD with scalars is demonstrated. The model Lagrangian has the same symmetry properties as QCD. The effective potential for the "field" M is obtained in an instanton background and is shown to have a non trivial minimum. This expectation value generates masses for the quarks due to the breakdown of chiral symmetry. The method of Raby of introducing tadpole terms to break the $SU(3)$ flavor symmetry is realized here by the minimization condition with $m_u = m_d \neq m_s$.

The crucial role of the effective potential (9) can be appreciated by comparing it with the situation where there are no instantons, like that of the Saviddy background field, for example. In this case one introduces a mass term for the quarks, evaluates the one-loop effective potential and demands a minimum lower than zero when the mass of the quark goes to zero. This is known not to produce chiral symmetry breaking since $V_{eff} \sim \alpha m^4 + \beta m^4 \ln(m^2/\mu^2)$ whose minimum value is zero when $m^2 \to 0$. In our case, the effective potential is given by (9) and none of the λ's can equal zero since $\det M \neq 0$ always. (In the case of $SU(2)$ for example, $\det M = -(\phi_1^2 + \phi_2^2 + \phi_3^2)$ for $M = \phi^a\tau^a$). So the analogous step of $m \to 0$ does not arise in our case. The introduction of elementary scalars through a σ-model type of interaction with scalars and pseudoscalars gave rise to the effective potential (9). The minimisation of this gives the VEV for M and when M is expanded about this VEV, produces a mass term for the quark through the Yukawa coupling in (2). Thus a vacuum of the theory that is not chirally invariant (as dictated by QCD sum rules and other studies) has been obtained.

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